Generalizing DBH and height prediction in coast Douglas-fir and red alder

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Forest Inventory: Traditional

- Driven by ground measurement
- Method: Sampling (dominated by StRS)

- DBH: main dendrometric attribute
- Derived attributes:
  - Height
  - Volume

Forest Inventory: Next Generation

- Driven by remote sensing
- Method: population or sophisticated sampling

- Total height: main dendrometric attribute
  - Projected crown area
- Derived attributes:
  - DBH
  - Volume

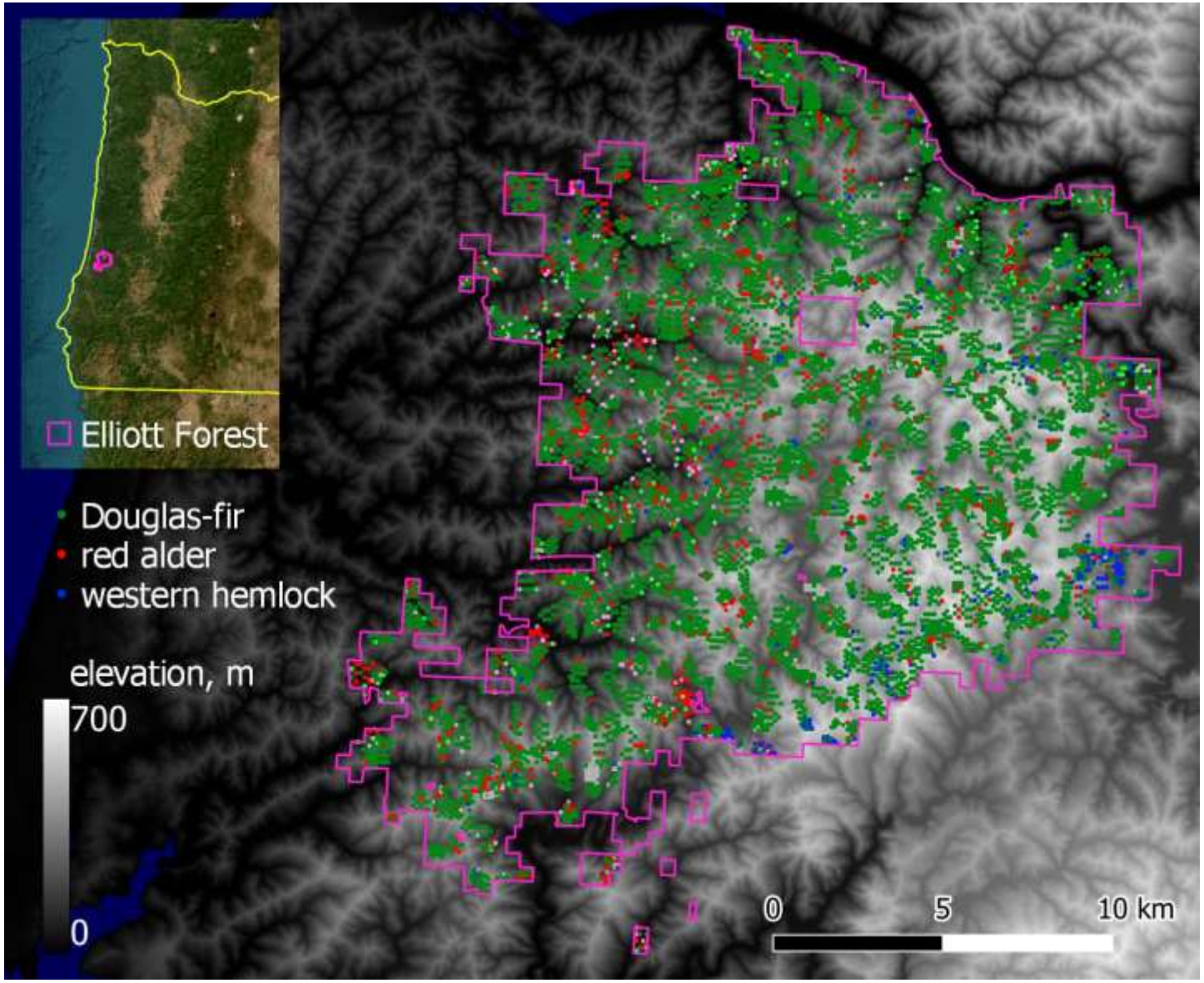
Objective

- Tree-level DBH-Height equations for Douglas fir and red alder
  - Two major Pacific Northwest species

- Focus on:
  - Accuracy and parsimony
  - Coherence: simultaneous and invertible relationships
  - Computational efficiency

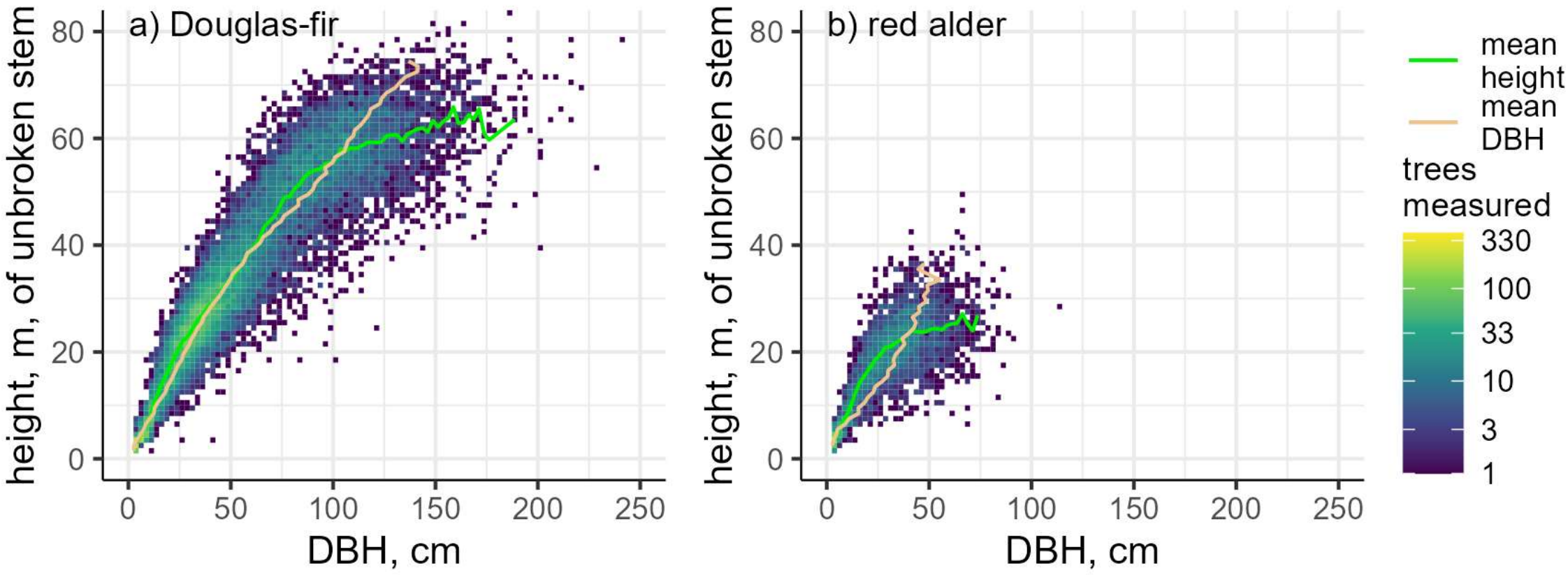
Study Area: Elliott State Research Forest

- Coastal rainforest
  - ~33,000 ha
  - 70–110% slopes
  - 1900 – 2800 mm H_2O yr^{-1}
  - T_{annual} 10–12 °C
- Complex history
  - Multifire stands (1800s)
  - Plantations (1950+)



Data

- 2016 ground inventory
  - 18,363 plots: 97,424 trees with 33,447 height measure trees



Models

- Nonlinear regression
  - 10 DBH forms generalized
  - 13 height forms generalized
- Generalized Additive Models (GAMs)
  - One form each for DBH and Height
- Linear regression
  - Linear and parabolic controls

$$DBH = a_1(H - 1.37)^{b_1} e^{b_2(H-1.37)}$$

$$H = 1.37 + a_1 H_{100}{}^{b_1} \left(1 - e^{b_2 \left(\frac{TPH}{BA}\right)^{b_3} DBH}\right)^{b_4}$$

$$DBH = a_0 + \sum_i a_i f_i(H)$$

$$DBH = a_1(H - 1.37)$$

$$DBH = a_1(H - 1.37) + a_2(H - 1.37)^2$$

GAMs [in this study]

- Penalized thin plate splines
  - Constrained flexibility
  - Fit using REML
- Nonparametric controls
  - Assess structural error
- Can be computationally intensive

Generalizing Predictors

- Stand history
  - natural regeneration, plantation
- Physiography
  - physio = elevation + slope + aspect + TSI
- Relative height and relative DBH
  - RelHt = H / H_{100}, RelDbh = DBH / QMD
- Basal area
  - BA+L = basal area (BA) + basal area larger than current tree (BAL)
  - ABA+T = approximate BA (ABA) + basal area taller (AAT)

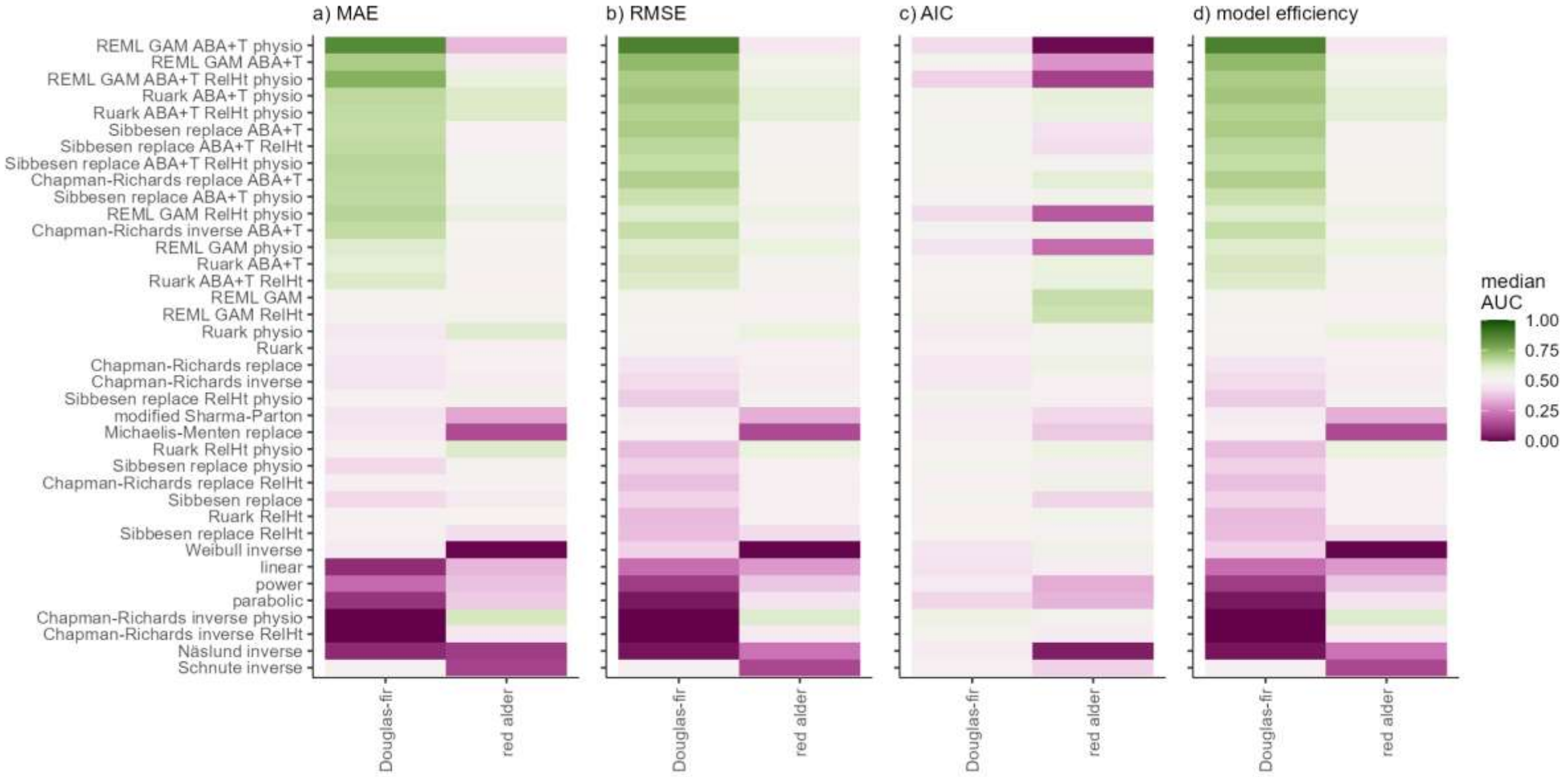
Model Assessment

- Cross validated ROC AUC
  - Rank models by predictive ability (0 = least accurate, 1 = most)

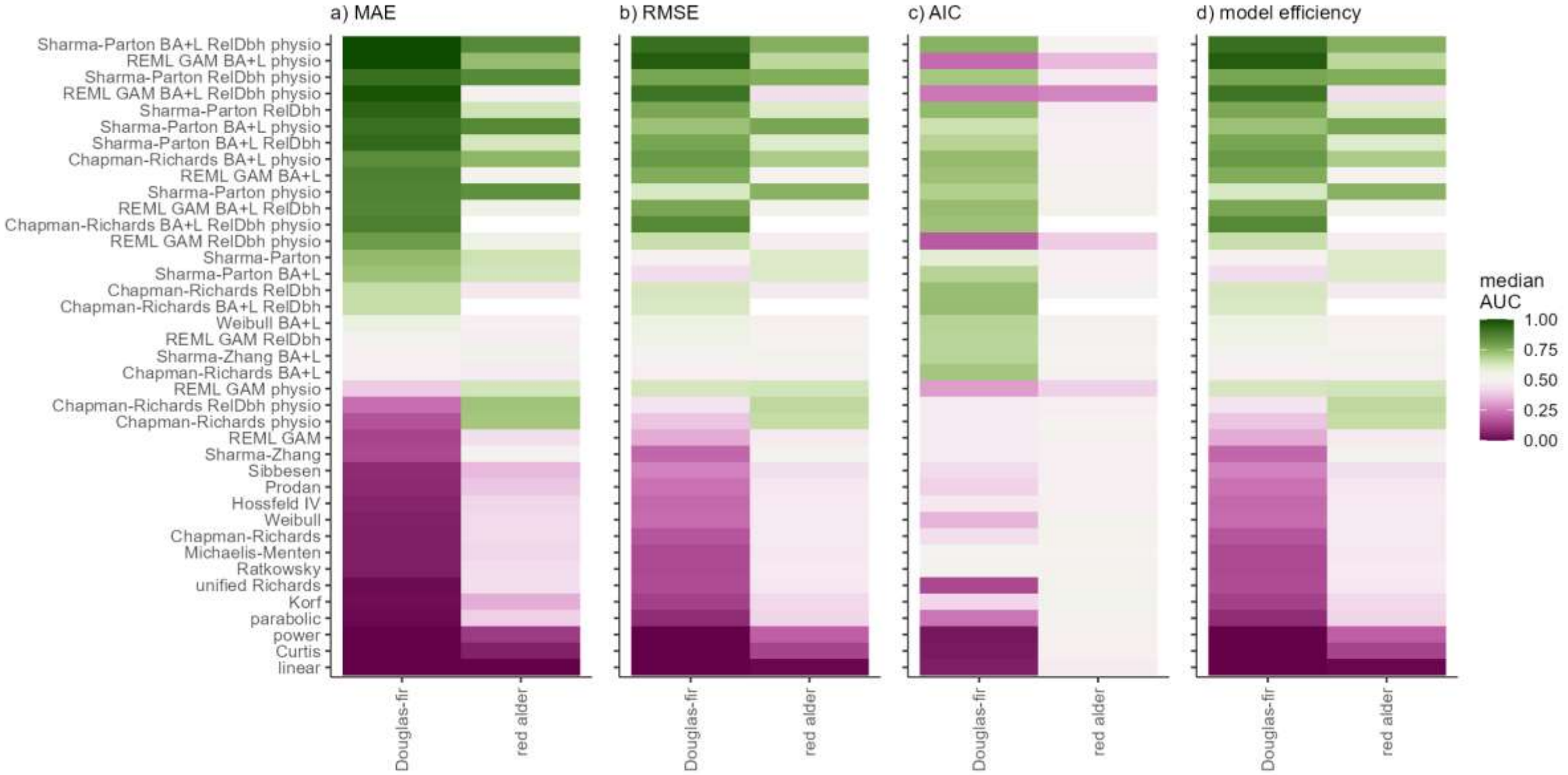
- Goodness of fit
  - MAE, RMSE, AIC, model efficiency
  - Nash-Sutcliffe definition

$$efficiency = 1 - \frac{\sum(Obs - Model)^2}{\sum\left(Obs - \overline{Obs}\right)^2}$$

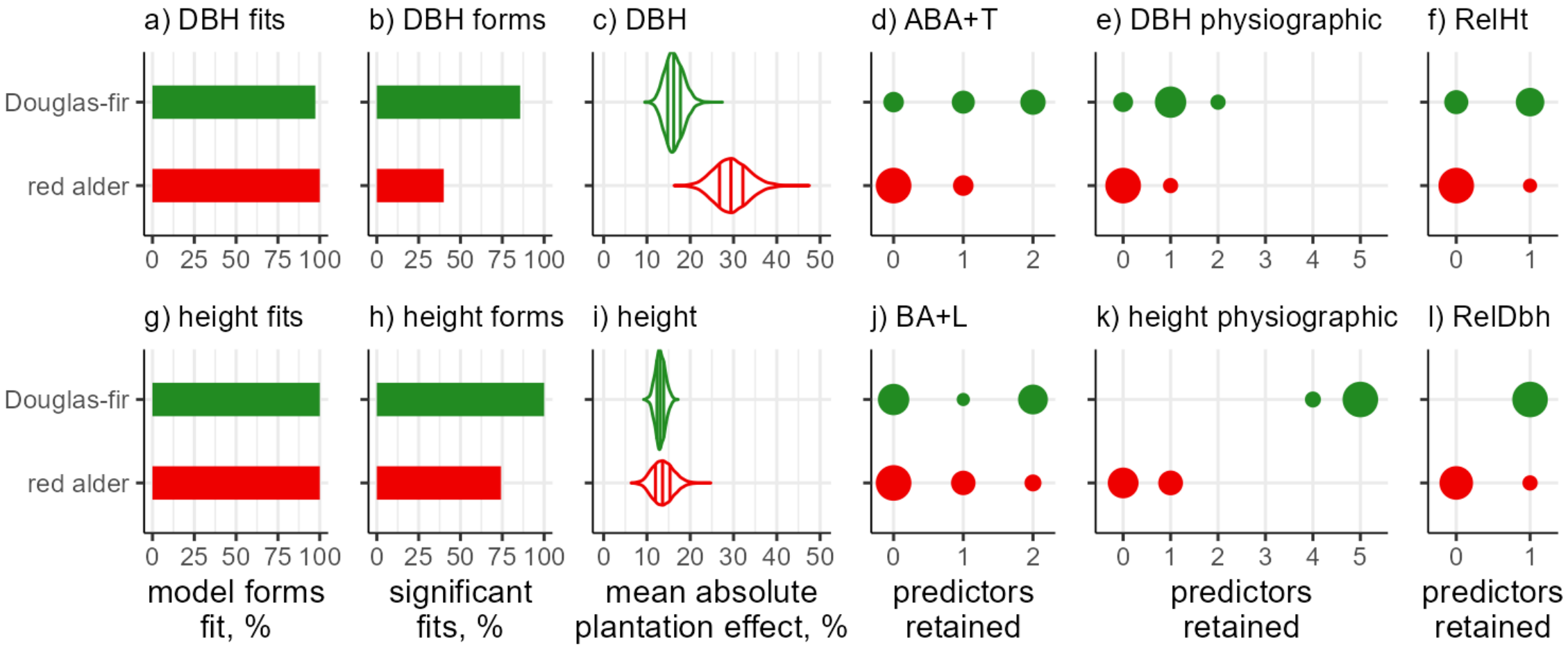
Results: DBH


a) MAE
b) RMSE
c) AIC
d) model efficiency
REML GAM ABA+T physio
REML GAM ABA+T
REML GAM ABA+T RelHt physio
Ruark ABA+T physio
Ruark ABA+T RelHt physio
Sibbesen replace ABA+T
Sibbesen replace ABA+T RelHt
Sibbesen replace ABA+T RelHt physio
Chapman-Richards replace ABA+T
Sibbesen replace ABA+T physio
REML GAM RelHt physio
Chapman-Richards inverse ABA+T
REML GAM physio
Ruark ABA+T
Ruark ABA+T RelHt
REML GAM
REML GAM RelHt
Ruark physio
Ruark
Chapman-Richards replace
Chapman-Richards inverse
Sibbesen replace RelHt physio
modified Sharma-Parton
Michaelis-Menten replace
Ruark RelHt physio
Sibbesen replace physio
Chapman-Richards replace RelHt
Sibbesen replace
Ruark RelHt
Sibbesen replace RelHt
Weibull inverse
linear
power
parabolic
Chapman-Richards inverse physio
Chapman-Richards inverse RelHt
Näslund inverse
Schnute inverse
Douglas-fir
red alder
median AUC
1.00
0.75
0.50
0.25
0.00

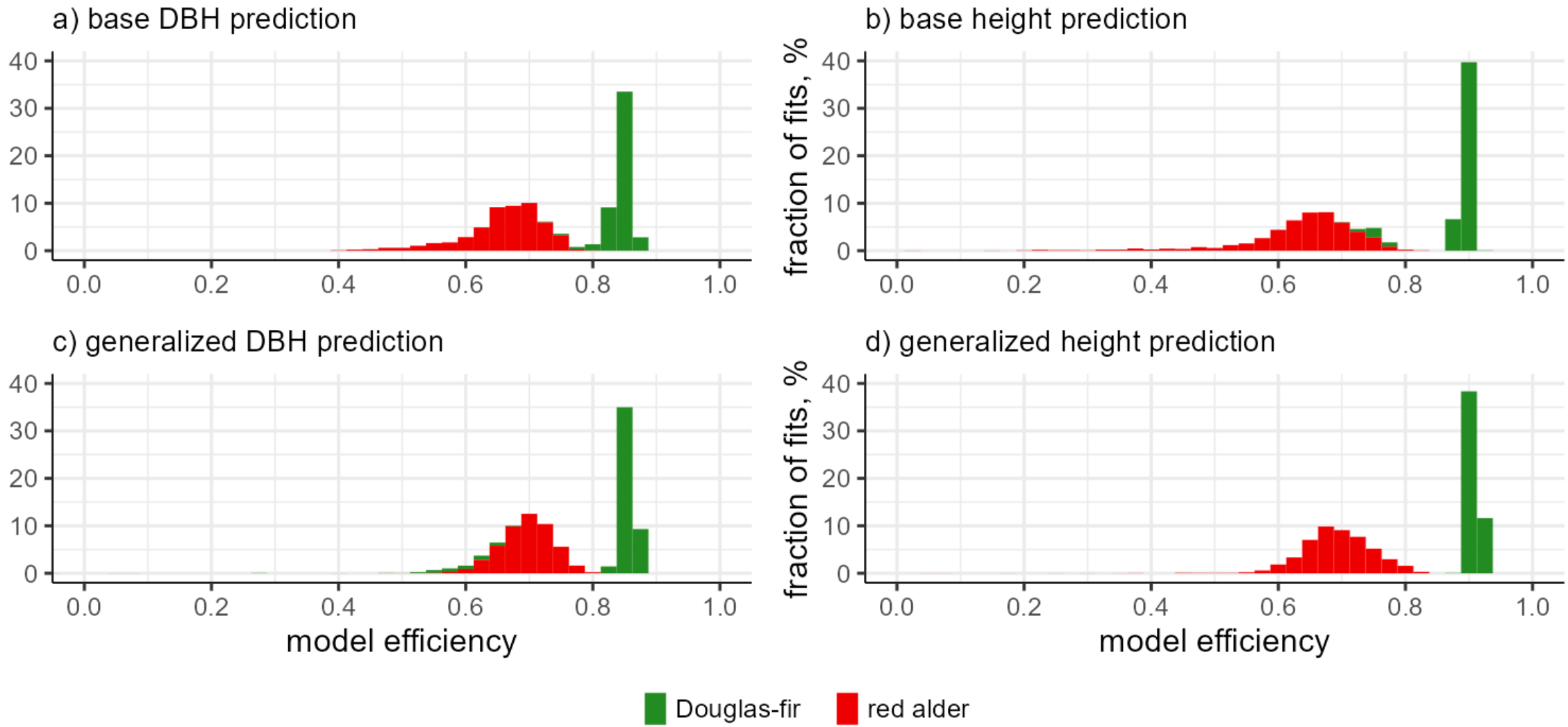
Results: Height

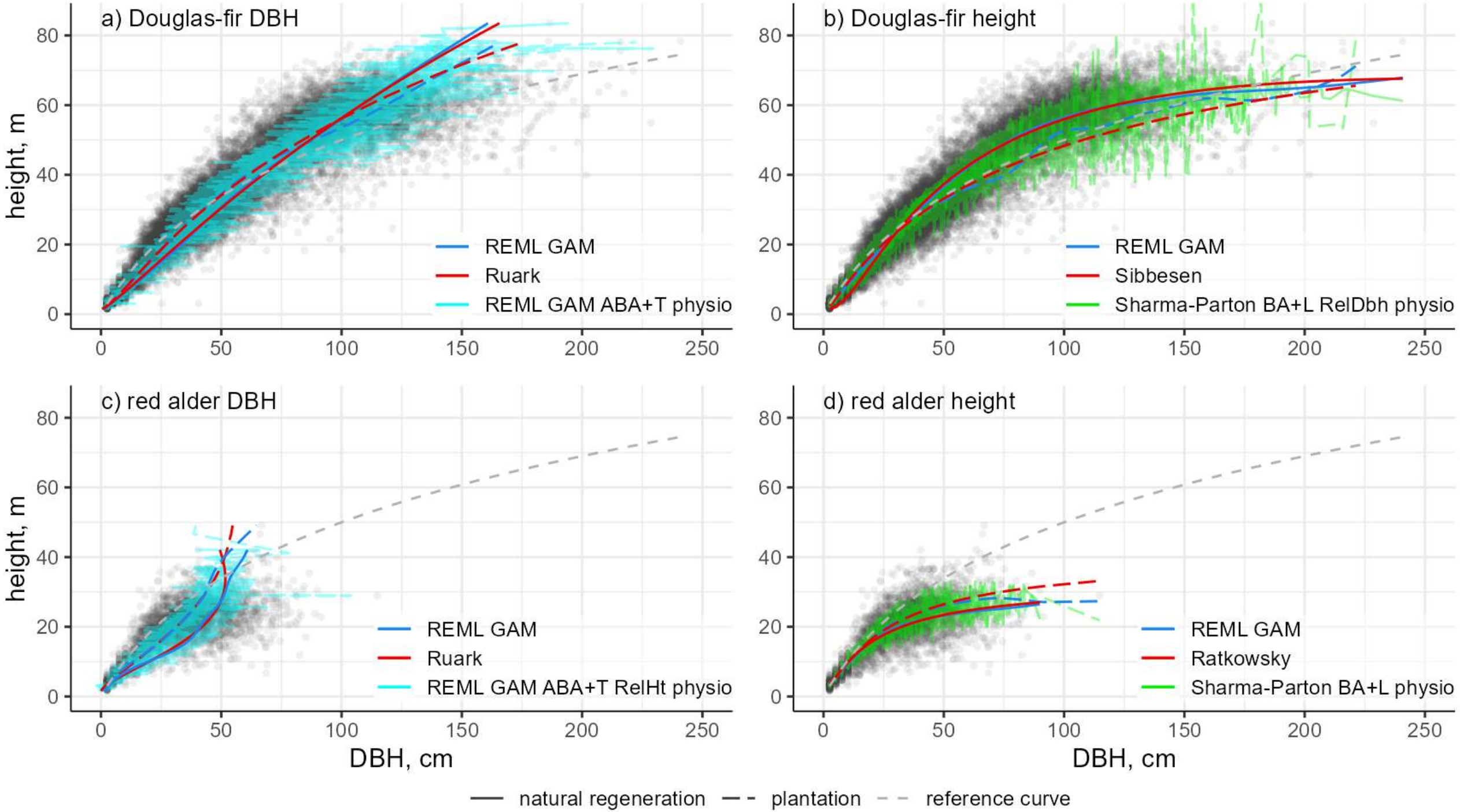


Results: Forms and Predictors



Results: Accuracy




a) Douglas-fir DBH
REML GAM
Ruark
REML GAM ABA+T physio
b) Douglas-fir height
REML GAM
Sibbesen
Sharma-Parton BA+L RelDbh physio
c) red alder DBH
REML GAM
Ruark
REML GAM ABA+T RelHt physio
d) red alder height
REML GAM
Ratkowsky
Sharma-Parton BA+L physio
height, m
DBH, cm
natural regeneration
plantation
reference curve

Main Findings

- Species dependent model efficiency
  - > 0.85 for Douglas-fir
  - > 0.70 for red alder
- Nonlinear regression form
  - Height: Sharma-Parton
  - DBH: Ruark
- Prediction of 1 million trees
  - Nonlinear: < 1 sec
  - GAM: > 1 minute (base GAM)

    > 10 minutes (complex GAM)

Conclusion

- Control forms are valuable
  - GAMs can be more accurate (not by much)
  - Simple linear regression sometimes wins (for other species)

- Chapman-Richards
  - Not as accurate as others
  - Harder to fit
  - The ONLY One Invertible: Height and DBH

$$H = 1.37 + a_1\left(1 - e^{b_1 DBH}\right)^{b_2} \qquad DBH = a_1 \log\left(1 - \min\left(b_1(H - 1.37)^{b_2}, l_1\right)\right) \qquad DBH = a_1\left(1 - e^{b_1(H-1.37)}\right)^{b_2}$$

Unexpected result

1. Invertibility is not consistent:
   - Functional or Mathematical

2. Complements the Incompleteness Theorem of Gödel
   - Not rooted in Aristotelian logic weakness: ”I am lying”

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